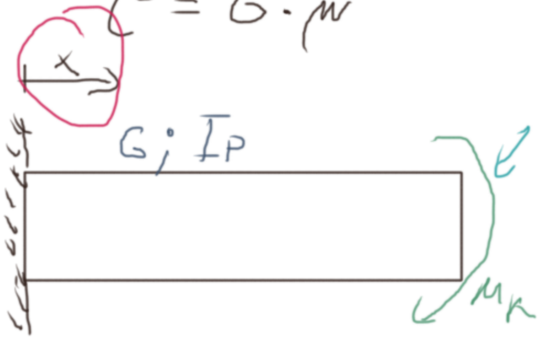
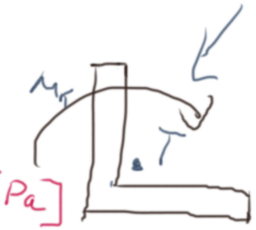
NAMÁHÁNÍ KRUTEM

– PRO KRUHOVÉ PRŮŘEZY

→ COULOMBŮV ZÁKON

$$\tau = G \cdot \gamma$$

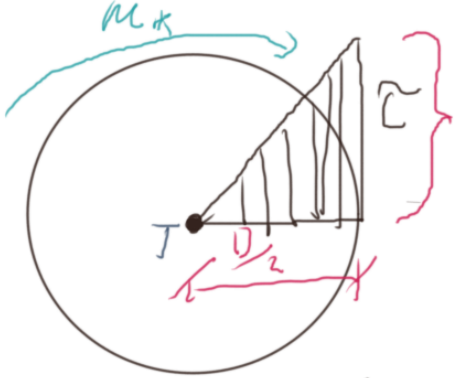




τ – SMYKOVÉ NAPĚTÍ [Pa]

G – SMYKOVÝ MODUL [Pa]

γ – ÚHEL ZKROUCENÍ [RAD]



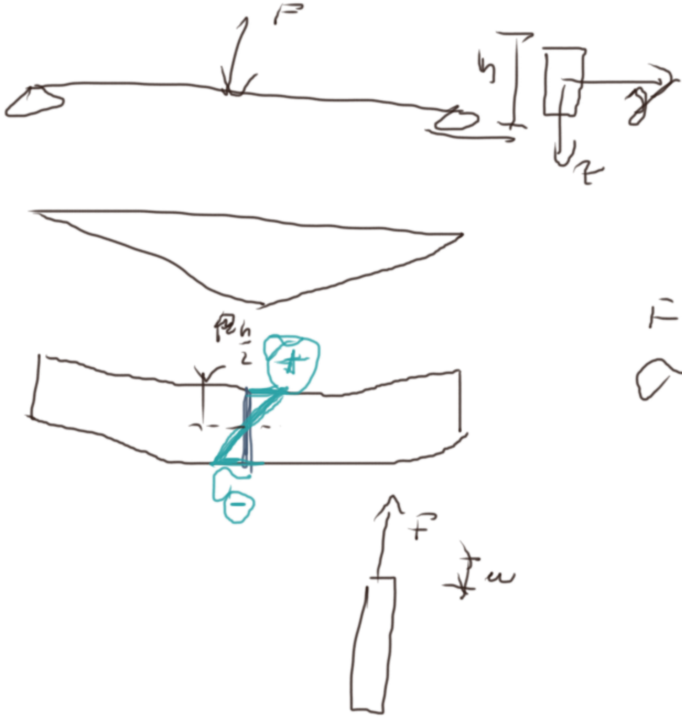
NAPĚTÍ PŘI KRUTU

$$\tau(x;\rho) = \frac{M_K(x)}{I_P(x)} \cdot \rho$$

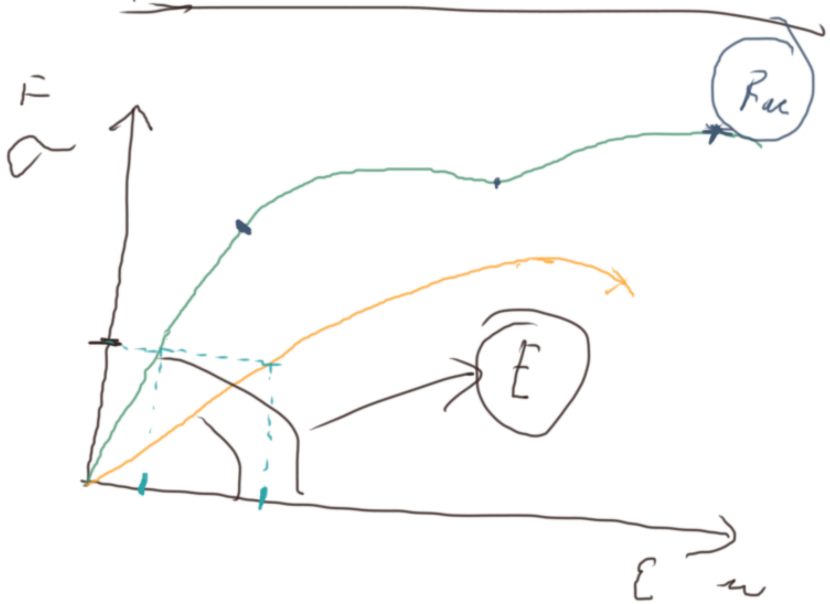
$$\tau_P \geq \tau_{max} = \frac{M_{Kmax}}{I_P} \cdot \frac{D}{2}$$

M_K – KROUTÍCÍ MOMENT [N·m]

I_P – POLÁRNÍ MOMENT SETRVAČNOSTI



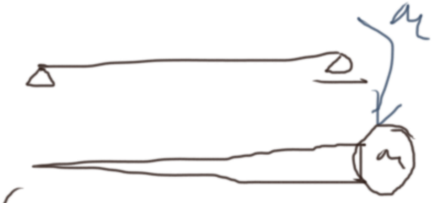
$$\sigma = \frac{M_{max}}{I_y} \cdot \frac{h}{2}$$



DEFORMAČNÍ ENERGIE

$$\lambda_k = \frac{1}{2}\,\tau\cdot\mu$$

$$E = \int_V \lambda_k \, dV = \frac{1}{2G\cdot I_p}\int_{(L)} M_k^2(x)\,dx \qquad E = \frac{1}{2E\cdot I_y}\int_{(L)} M^2(x)\,dx$$

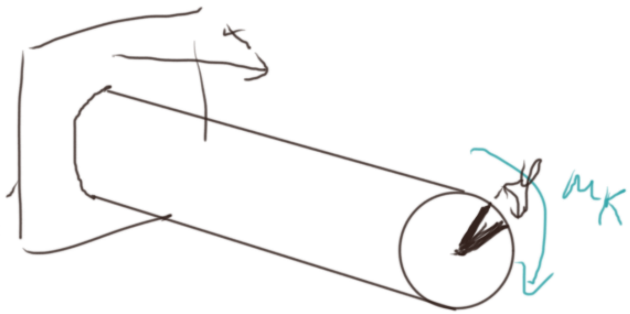


FUNKCE ÚHLU ZKROUCENÍ

$$\varphi(x) = \frac{1}{G I_p}\int M_k(x)$$

POMĚRNÝ ÚHEL ZKROUCENÍ

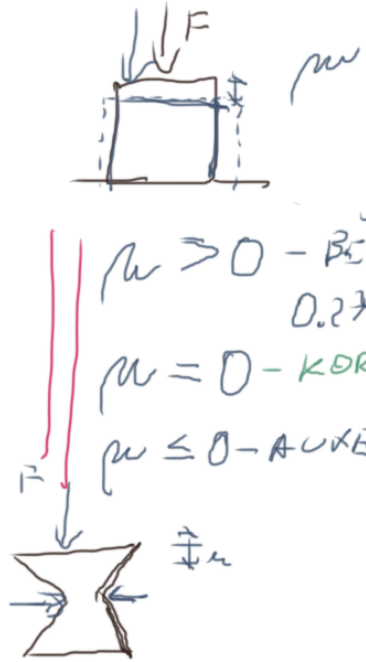
$$\Delta = \frac{d\varphi}{dx}$$



YMP

$E \; G \rightarrow$

$$G = \frac{E}{2(\mu+1)}$$

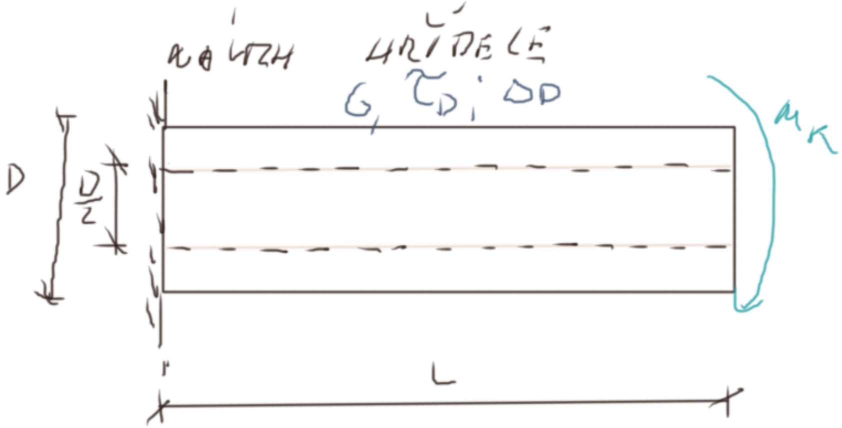


$\mu > 0$ – BĚŽNÝ M

0,27 – 0,3

$\mu = 0$ – KOREK

$\mu \leq 0$ – AUXETIC



DÁNO: L

M_K

G

τ_D — PEVNOSTNÍ PODMÍNKA

Δ_D — TUHOSTNÍ PODMÍNKA

D — NEZNÁMÁ

$$I_p = \frac{\pi}{32} \cdot D_\tau^4 - \frac{\pi}{32}\left(\frac{D_\tau}{2}\right)^4 = \frac{15\pi}{512} D_\tau^4$$

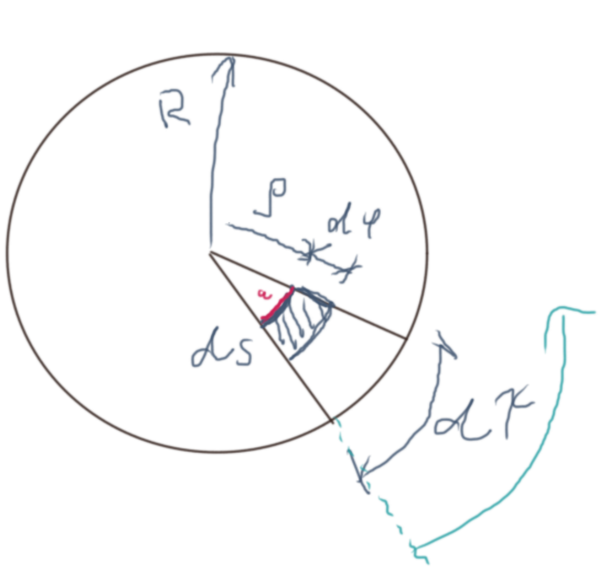
PEVNOSTNÍ PODMÍNKA

$$\tau_D \geq \tau = \frac{M_K}{I_p} \cdot \frac{D_\tau}{2}$$

$$\tau = \frac{M_K}{\frac{15\pi}{512} D_\tau^4} \frac{D_\tau}{2} = \frac{256 \cdot M_K}{15\pi \cdot D_\tau^3} \rightarrow D_\tau \geq \sqrt[3]{\frac{256\, M_K}{15\pi\tau}}$$

$$\Delta_D \geq \Delta = \frac{M_K}{G \cdot I_p} = \frac{512 M_K}{15\pi \cdot G \cdot D_\Delta^4} \rightarrow D_\Delta \geq \sqrt[4]{\frac{512\, M_K}{15\pi G \cdot \Delta}}$$

$$D = \max\{D_\tau ; D_\Delta\}$$



$$I_p = \int_{(S)} \rho^2 \cdot dS$$

$$d\varphi = \frac{a}{\rho}$$

$$dS = a \cdot d\psi$$

$$dS = d\varphi \cdot \rho \cdot d\psi$$

$$dS = \rho \cdot d\varphi \cdot d\rho$$

$$I_p = \int_{(S)} \rho^3 \cdot \underbrace{d\varphi} \cdot \underbrace{d\rho}$$

$$R = \frac{D}{2}$$

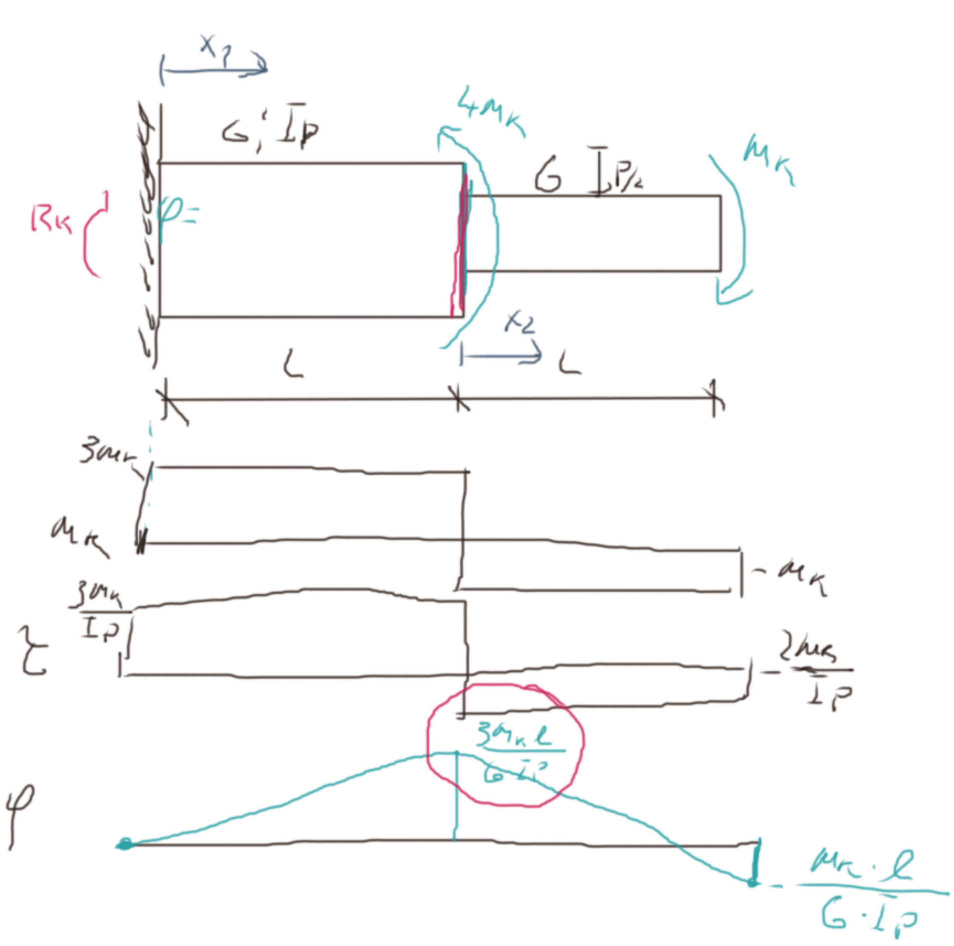
$$I_p = \int_0^R \rho^3 d\rho \int_0^{2\pi} d\varphi = \frac{R^4}{4} \cdot 2\pi = \frac{1}{32}\pi D^4$$

$R_K = 4M_K - M_K = 3M_K$

$$I_{P/2} = \frac{I_P}{2}$$

1) MAXIMÁLNÍ ÚHEL ZKROUCENÍ

2) CELKOVÁ DEF. ENERGIE



$$\varphi(x) = \int \frac{M_k(x)}{G I_P}\,dx$$

$x_1 \in \langle 0; l)$

$$\varphi(x_1) = \int \frac{3M_k}{G I_P}\,dx_1 = \frac{3M_k}{G I_P} x_1 + C_1$$

$\varphi(0) = 0 \rightarrow C_1 = 0$

$$\varphi(x_1) = \frac{3M_k}{G I_P}\cdot x_1 \qquad \varphi(l) = \frac{3M_k \cdot l}{G \cdot I_P}$$

$x_2 \in \langle 0; l)$

$$\varphi(x_2) = \int \frac{-2M_k}{G \cdot I_{P/2}}\,dx_2 = -\frac{2M_k}{G I_{P/2}} \cdot x_2 + C_2$$

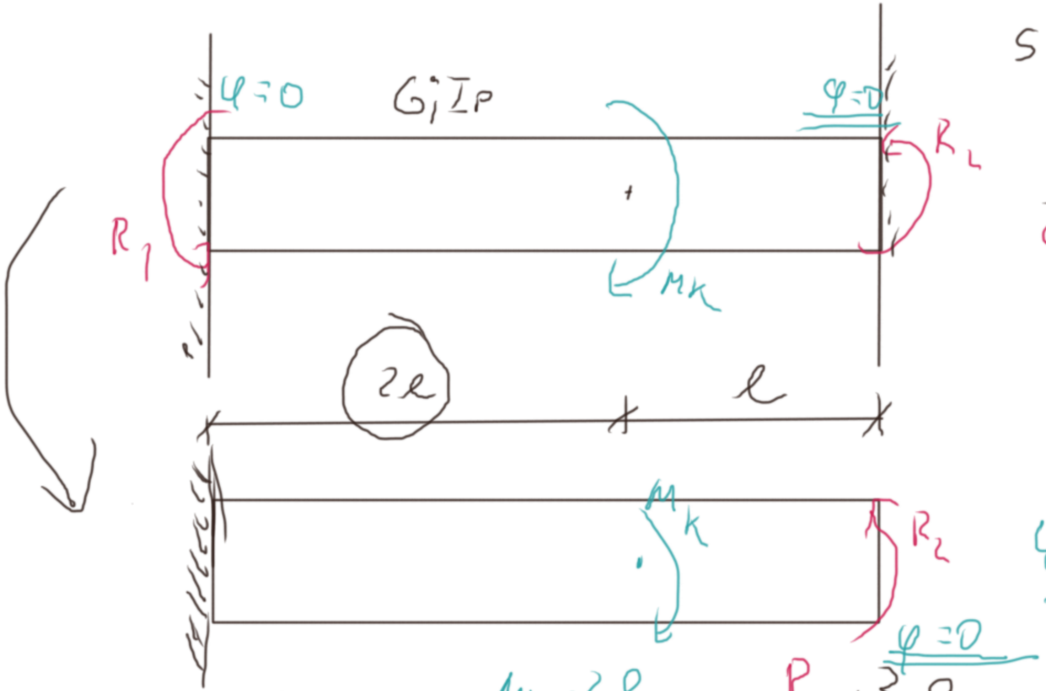
$\varphi(0) \quad C_2 = \frac{3M_k \cdot L}{G \cdot I_P}$

$$\varphi(x_2) = -\frac{4M_k}{G I_P} x_2 + \frac{3M_k}{G I_P}$$

$$\varphi(l) = -\frac{M_k \cdot L}{G I_P} \leftarrow$$

$$E = \frac{1}{2GI_p}\int_{(l)} M_k^2(x)\,dx = \frac{1}{2GI_p}\int_0^l (3M_k)^2\,dx + \frac{1}{2G\cdot I_{p/2}}\int_0^l (-M_k)^2\,dx =$$

$$= \frac{9M_k^2 l}{2GI_p} + \frac{M_k^2 l}{GI_p} = \boxed{\frac{11M_k^2\cdot l}{2G\cdot I_p}}$$



STANOVTE REAKCE R_1 a R_2

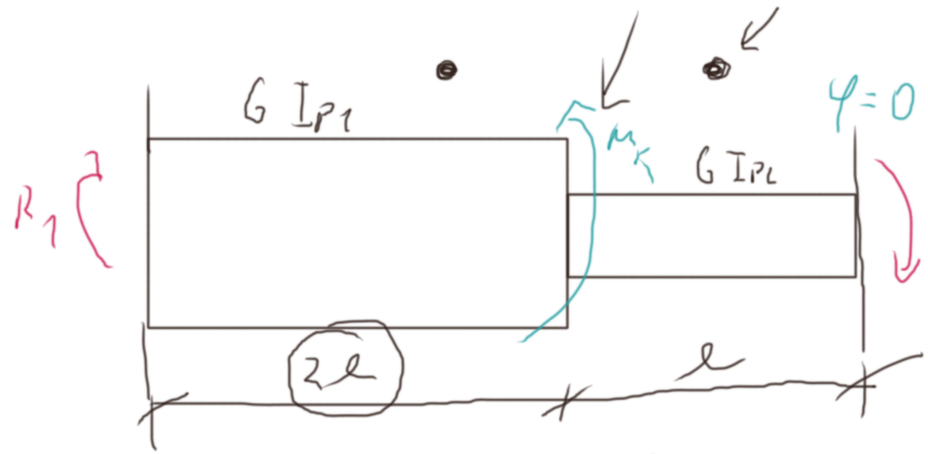
STATICKÉ PODMÍNKY

$$R_1 - M_K + R_2 = 0$$

$$\varphi(3l) = 0$$

$$\varphi(3l) = 0 = \frac{M_K \cdot 2l}{G I_p} - \frac{R_2 \cdot 3l}{G I_p} = 0 \longrightarrow R_2 = \frac{2}{3} M_K$$

$$\boxed{R_1 = \frac{1}{3} M_K}$$



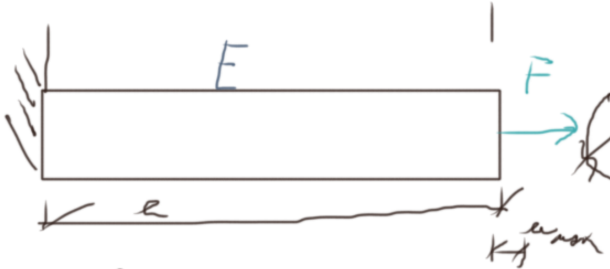
$$R_1 + R_2 - M_K = 0$$

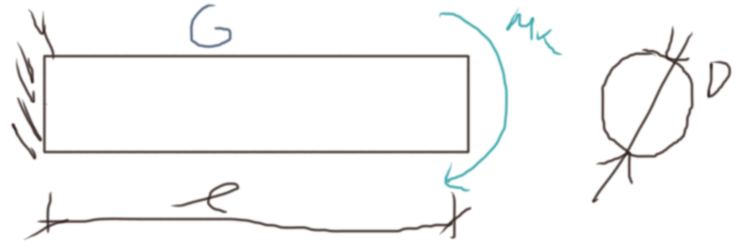
PODDAJNOST TORZNI

δ

$$\varphi(3l) = 0 = \frac{M_K \cdot 2l}{G \cdot I_{p1}} - R_2 \left(\frac{l}{G I_{p2}} + \frac{2l}{G I_{p1}} \right) = 0$$

$$\frac{1}{\delta} = k$$ – TUHOST TORZNI





$$u(x) = \int \frac{F}{E \cdot A}\, dx$$

$$\varphi(x) = \int \frac{M_K}{G \cdot I_p}\, dx$$

$$u(l) = \frac{F l}{E \cdot A}$$

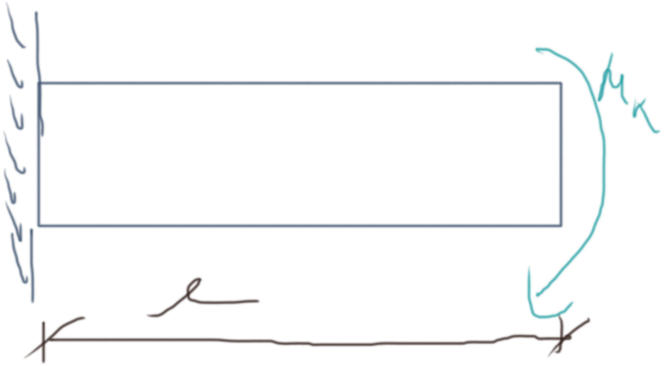
$$\varphi(l) = \frac{M_K \cdot l}{G\, I_p}$$

u

φ

$$U = \int_0^l \frac{F^2}{2 E A} = \frac{1}{2} \frac{F^2 l}{E \cdot A}$$

$$U = \int_0^l \frac{M_K^2}{2 \cdot G I_p} = \frac{1}{2} \frac{M_K^2\, l}{G \cdot I_p}$$



$$\boxed{\varphi_{max} = \frac{M_K\, l}{G\, I_p}}$$

$$U = E = \frac{1}{2 G I_p} \int_0^l M_K^2\, dx = \boxed{\frac{M_K^2\, l}{2 G I_p}}$$

$$\boxed{W = \frac{1}{2} M_K \cdot \varphi} \longrightarrow U = W$$

$$\frac{M_K^{\cancel{2}}\, l}{\cancel{2} G I_p} = \frac{1}{\cancel{2}} \cancel{M_K} \cdot \varphi \longrightarrow \boxed{\varphi = \frac{M_K \cdot l}{G \cdot I_p}}$$