

$$x \in (0; l) \\ M(x) = -q \cdot x \cdot \frac{x}{2}$$

$$-E \cdot I_y \cdot w''(x) = -q \cdot \frac{x^1}{2}$$

$$EI_y w'(x) = \frac{q x^3}{6} + C_1$$

$$EI_y w(x) = \frac{q x^4}{24} + C_1 x + C_2 \quad \leftarrow w(x)$$

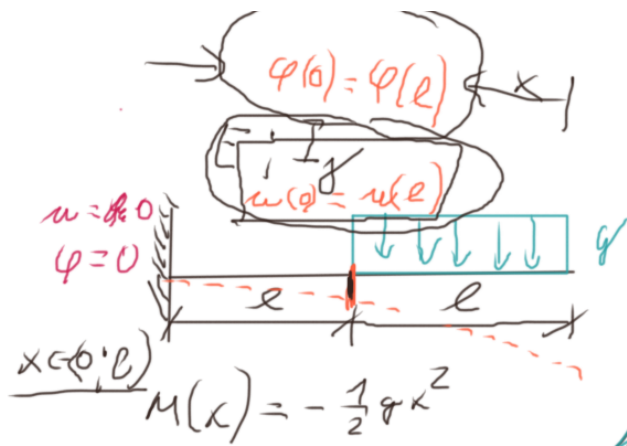
$$w(l) = 0 \rightarrow 0 = \frac{q l^3}{6} + C_1 \rightarrow C_1 = -\frac{q l^3}{6}$$

$$w(l) = 0 \rightarrow 0 = \frac{q l^4}{24} - \frac{q l^3}{6} \cdot l + C_2 \rightarrow C_2 = \frac{1}{8} q l^4$$

$$w(x) = \frac{1}{EI_y} \left( \frac{q x^4}{24} - \frac{q l^3}{6} \cdot x + \frac{1}{8} q l^4 \right)$$

$$\varphi(x) = \frac{1}{EI_y} \left( \frac{q x^3}{6} - \frac{q l^3}{6} \right)$$

$w_{max} = \frac{1}{8} \frac{q l^4}{EI_y}$  |  $\varphi_{max} = \frac{q l^3}{6 EI_y}$



$w(x)$   
 $\varphi(x)$

$$-1 + 2 = 1$$

$$-1 + \frac{4}{3} = 1 \quad -\frac{8}{6} + \frac{4}{3} = -\frac{4}{3}$$

$$+EI_y w''(x) = \frac{1}{2} q x^2$$

$$EI \varphi(x) = \frac{q x^3}{6} + C_1$$

$$EI w(x) = \frac{q x^4}{24} + C_1 x + C_2$$

$$\varphi(l) = 4l$$

$$\frac{q l^3}{6} + C_1 = -\frac{1}{2} q l^3 + \frac{q l^3}{2} - q l^3$$

$$C_1 = -q l^3 \cdot \frac{1}{6} q l^3 = -\frac{7}{6} q l^3$$

$$w(l) = w(l)$$

$$\frac{q l^4}{24} - \frac{7}{6} q l^4 + C_2 = -\frac{1}{4} q l^4 + \frac{1}{6} q l^4 - q l^4 + \frac{4}{3} q l^4$$

$$C_2 = \left( \frac{1}{24} + \frac{1}{6} - 1 + \frac{4}{3} - \frac{1}{24} + \frac{7}{6} \right) q l^4 = \frac{-6 + 4 - 24 + 32 - 1 + 28}{24} q l^4 = \frac{33}{24} q l^4$$

$x \in (l; 2l)$

$$M(x) = -q l \left( x - \frac{l}{2} \right) = -q l x + \frac{1}{2} q l^2$$

$$EI_y \cdot w''(x) = -\frac{1}{2} q l^2 + q l x$$

$$EI_y \cdot \varphi(x) = -\frac{1}{2} q l^2 x + \frac{q l x^2}{2} + C_3$$

$$EI_y w(x) = -\frac{1}{4} q l^2 x^2 + \frac{q l x^3}{6} + C_3 x + C_4$$

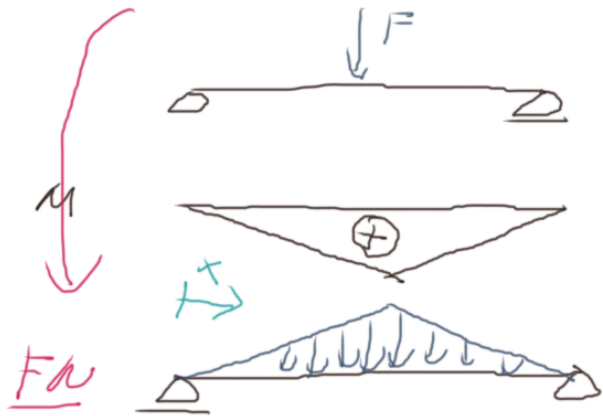
$$\varphi(2l) = 0 \rightarrow 0 = -\frac{1}{2} q l^2 \cdot 2l + \frac{1}{2} q l \cdot 4l^2 + C_3$$

$$C_3 = -q l^3$$

$$w(2l) = 0 \rightarrow 0 = -\frac{1}{4} q l^2 \cdot 4l^2 + \frac{1}{6} q l \cdot 8l^3 - q l^4 + C_4$$

$$C_4 = \frac{4}{3} q l^4$$

# MOHROVA METODA



- 1) ZJISTIŤ PRŮBĚH ODPISOVÁČO M
- 2) FIKTIVNÍ NOSNÍK
- 3) ZATÍŽENÍ  $0M$  ZE SKUTIČNÍHO NOSNÍKU

→ rovnice

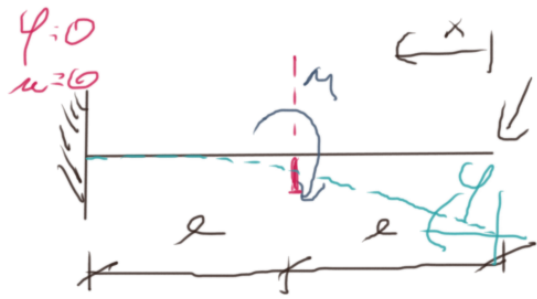
$$\varphi(x) = \frac{T_F(x)}{EI}$$
$$u(x) = \frac{M(x)}{EI}$$

SN



FN





$$x \in (0; l) \quad \text{①}$$

$$m(x) = 0$$

$$-EI_y w''''(x) = 0$$

$$-EI_y w'''(x) = C_1$$

$$-EI_y w''(x) = C_1 \cdot x + C_2$$

$$EI_y w'(x) = -C_1 x - C_2$$

$$w(l) = w(l)$$

$$-C_1 = Ml - 2Ml$$

$$C_1 = M \cdot l$$

$$w(l) = w(l)$$

$$-C_1 x - C_2 = M \frac{x^2}{2} + C_3 x + C_4$$

$$-Ml^2 - C_2 = M \frac{l^2}{2} - 2Ml^2 + 2Ml^2$$

$$x \in (l; 2l) \quad \text{②}$$

$$m(x) = -M$$

$$EI_y w''''(x) = M$$

$$EI_y w'''(x) = Mx + C_3$$

$$EI_y w''(x) = M \frac{x^2}{2} + C_3 x + C_4$$

$$w(l) = 0 \rightarrow 0 = M \cdot l + C_3 \rightarrow C_3 = -2Ml$$

$$w(2l) = 0 \rightarrow 0 = M \cdot \frac{4l^2}{2} - 2Ml \cdot 2l + C_4$$

$$C_4 = 2Ml^2$$

$$C_2 = -\frac{3}{2} Ml^2$$

~~2Ml^2 + Ml^2~~

$$EI_y \varphi(x) = Ml$$

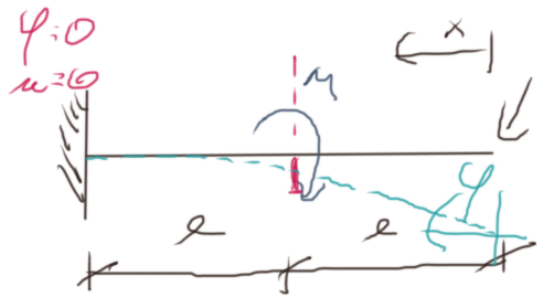
$$EI_y w(x) = -Ml \cdot x + \frac{3}{2} Ml^2$$

$$EI_y \varphi(x) = Mx - 2Ml$$

$$EI_y w(x) = M \frac{x^2}{2} - 2Ml x + 2Ml^2$$

~~$\varphi(0) = M \cdot l$~~

$$\varphi(0) = \frac{Ml}{EI_y}$$



$$x \in (0; l) \quad \text{①}$$

$$m(x) = 0$$

$$-EI_y w''''(x) = 0$$

$$-EI_y w'''(x) = C_1$$

$$-EI_y w''(x) = C_1 \cdot x + C_2$$

$$EI_y w'(x) = -C_1 x - C_2$$

$$w(l) = w(l)$$

$$-C_1 = Ml - 2Ml$$

$$C_1 = M \cdot l$$

$$w(l) = w(l)$$

$$-C_1 x - C_2 = M \frac{x^2}{2} + C_3 x + C_4$$

$$-Ml^2 - C_2 = M \frac{l^2}{2} - 2Ml^2 + 2Ml^2$$

$$x \in (l; 2l) \quad \text{②}$$

$$m(x) = -M$$

$$EI_y w''''(x) = M$$

$$EI_y w'''(x) = Mx + C_3$$

$$EI_y w''(x) = M \frac{x^2}{2} + C_3 x + C_4$$

$$w(l) = 0 \rightarrow 0 = M \cdot l + C_3 \rightarrow C_3 = -2Ml$$

$$w(2l) = 0 \rightarrow 0 = M \cdot \frac{4l^2}{2} - 2Ml \cdot 2l + C_4$$

$$C_4 = 2Ml^2$$

$$C_2 = -\frac{3}{2} Ml^2$$

~~2Ml^2 + Ml^2~~

$$EI_y \varphi(x) = Ml$$

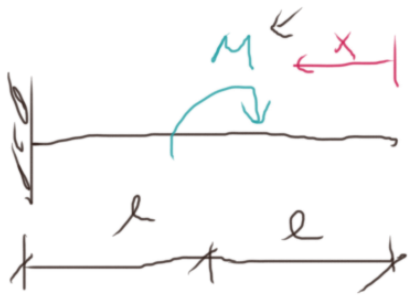
$$EI_y w(x) = -Ml \cdot x + \frac{3}{2} Ml^2$$

$$EI_y \cdot \varphi(x) = Mx - 2Ml$$

$$EI_y \cdot w(x) = M \frac{x^2}{2} - 2Ml x + 2Ml^2$$

~~$$\varphi(0) = M \cdot l$$~~

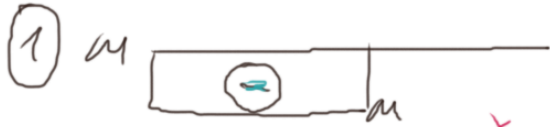
$$\varphi(0) = \frac{Ml}{EI_y}$$



$$\int_{F'} M_{F'}$$

$$\uparrow -A_F + M \cdot l = 0 \quad \leftarrow \quad M_{F_A} = M \cdot l \cdot \frac{3}{2} l = 0$$

$$A_F = M l \quad \quad \quad M_{F_A} = \frac{3}{2} M l^2$$



$$x \in (0; l)$$

$$T_F(x) = M l$$

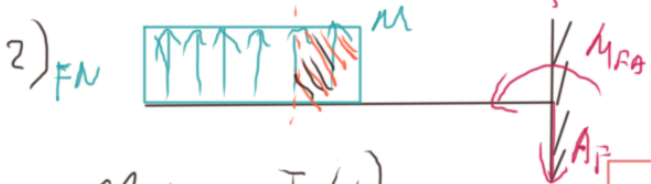
$$M_{F'}(x) = M_{F_A} - A_F \cdot x$$

$$= \frac{3}{2} M l^2 - M \cdot l \cdot x$$

$$x \in (l; 2l)$$

$$T_F(x) = M l - m \cdot (x - l)$$

$$M_{F'}(x) = \frac{3}{2} M l^2 - m l x + \frac{m(x-l)^2}{2}$$



$$T_F(x) = 2 M l - m x$$

$$M_{F'}(x) = \frac{m x^2}{2} - 2 M l x + 2 M l^2$$

- $\varphi(x) = \frac{T_F(x)}{E I_g}$
- $u(x) = \frac{M_{F'}(x)}{E I}$

$$E I_g \varphi(x) = M l$$

$$E I u(x) = \frac{3}{2} M l^2 - m l x$$

$$E I_g \varphi(x) = 2 M l - m x$$

$$E I u(x) = \frac{m x^2}{2} - 2 M l x + 2 M l^2$$

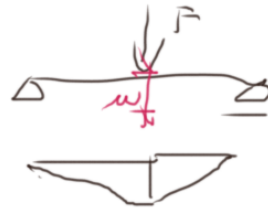
$$\varphi(0) = \frac{M \cdot l}{E I_g}$$

$$\rightarrow U = \int \frac{M^2(x)}{2 \cdot E I_y} dx = \frac{1}{2 E I_y} \left[ \int_0^l 0 dx + \int_l^{2l} M^2 dx \right] =$$

$$= \frac{1}{2 E I_y} \left[ M^2 x \right]_l^{2l} = \frac{1}{2 E I_y} (M^2 \cdot 2l - M^2 \cdot l) = \frac{M^2 l}{2 \cdot E I_y}$$

$$\frac{1}{2} M \cdot \varphi = \frac{M^2 l}{2 E I_y}$$

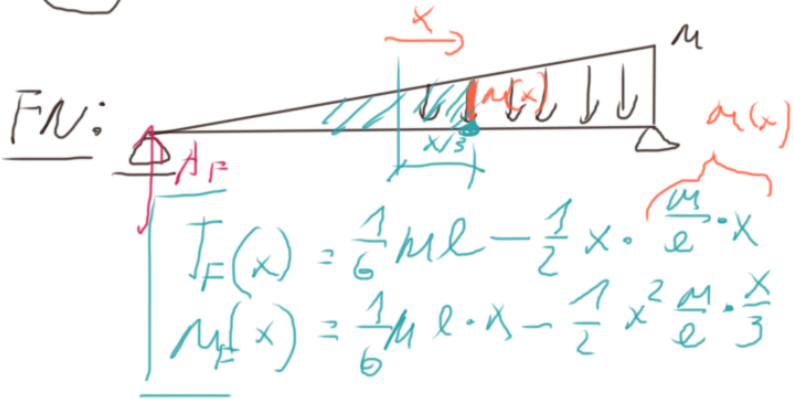
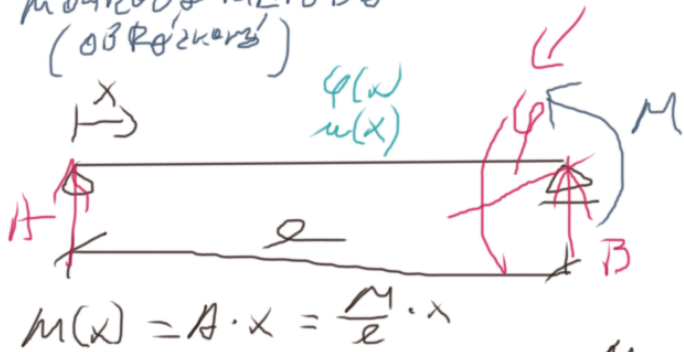
$$\varphi = \frac{M \cdot l}{E I_y}$$



$$\rightarrow \left( \frac{1}{2} F \cdot l \right) = \int \frac{M^2(x)}{2 E I_y} dx$$



МОМЕНТОВЫЙ МЕТОД  
(ОБРАЗЫВАЮЩИЙ)



$u(0) = 0$   
 $u(l) = 0$

$B \cdot l + m = 0 \rightarrow B = -\frac{M}{l}$   
 $A \cdot l - m = 0 \quad A = \frac{M}{l}$

$\frac{1}{6} - \frac{3}{6} = \frac{-2}{6} = -\frac{1}{3}$

$A_F \cdot l - \frac{1}{2} M \cdot l \cdot \frac{1}{3} l = 0 \rightarrow A_F = \frac{1}{6} M l$

$\frac{M(x)}{x} = \frac{M}{l} \quad m(x) = \frac{M}{l} \cdot x$   
 $\varphi(x) = \frac{1}{E I y} \left( \frac{1}{6} M l - \frac{1}{2} x^2 \frac{M}{l} \right)$   
 $u(x) = \frac{1}{E I y} \left( \frac{1}{6} M l \cdot x - \frac{1}{6} \frac{M}{l} \cdot x^3 \right)$

$\varphi(l) = \frac{1}{E I y} \left( \frac{1}{6} M l - \frac{1}{6} M l \right)$   
 $\varphi(l) = -\frac{1}{3 E I y} M l$



$$M(x) = A \cdot x$$

$$M(x) = \frac{M}{l} \cdot x$$

$w(0) = 0$

$$0 = -\frac{Mx^3}{6l} + C_1x + C_2 \Rightarrow C_2 = 0$$

$w(l) = 0$

$$0 = -\frac{Ml^3}{6l} + C_1 \cdot l$$

$$\frac{Ml^2}{6} = C_1 l \rightarrow C_1 = \frac{Ml}{6}$$

$$EI_y w''(x) = -\frac{M}{l} \cdot x$$

$$EI_y w'(x) = -\frac{Mx^2}{2l} + C_1$$

$$EI_y w(x) = -\frac{Mx^3}{6l} + C_1x + C_2$$

$$EI_y w'(x) = -\frac{Mx^2}{2l} + \frac{Ml}{6}$$

$$EI_y w(x) = -\frac{Mx^3}{6l} + \frac{Ml}{6}x$$

φ

$$U = \frac{1}{2EI} \int_0^l \frac{M^2 \cdot x^2}{EI} dx = \left[ \frac{m^2 \cdot x^3}{3EI} \right]_0^l = \frac{m^2 l^3}{3EI} = \frac{m^2 l}{3} \frac{1}{EI}$$

$$\frac{1}{2} M \cdot \varphi = \frac{1}{EI} \cdot \frac{m \cdot l}{3}$$

$$\varphi = \frac{ml}{3EI}$$