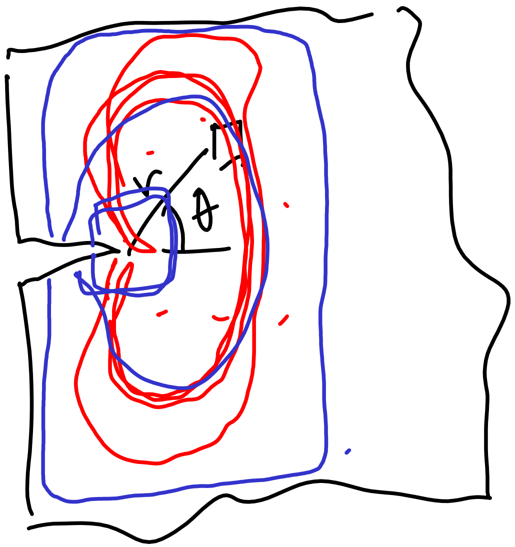
# NLM – Riceův J-integrál 1968

houževn. mat.

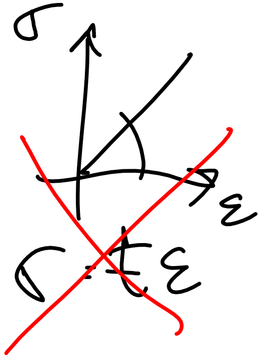
problém: vyjádřit napjatost v blízk. kořene trhliny




$\sigma(r, \theta)$

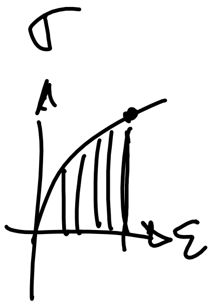
$r \to 0$

?

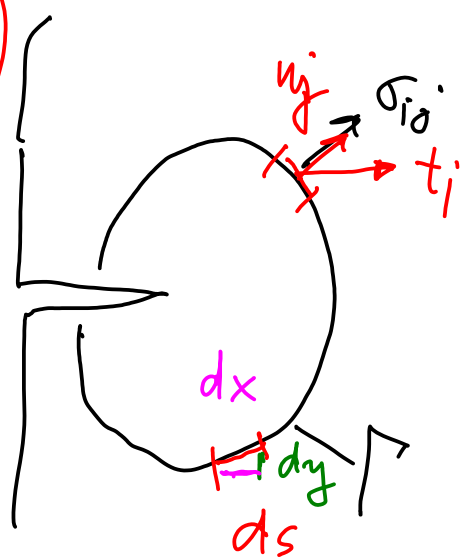


Ukázal, že J-int platí i pro plast. materiál, když je uvažováno monotónní zatěžování!

$$J = \int_{\Gamma} \left( \lambda \, dy - t_i \frac{\partial u_i}{\partial x} ds \right)$$




$$\lambda = \int \sigma \, d\varepsilon = \int_0^{\varepsilon} \sigma_{ij} \, d\varepsilon_{ij}$$




$t_i$ – vektor povrchových sil

$u_i$ – posunutí ve směru osy x

ds – element křivky

$$ds = \sqrt{dx^2 + dy^2}$$

vlastnosti J-integrálu:

a) nulová hodnota J-int po uzavřené křivce.

Green. věta: $$\oint_k (L\,dx + M\,dy) = \iint_A \left(\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}\right) dx\,dy$$

$L(x,y)$ a $M(x,y)$ - spojité 1. parc. der.

$$J = \iint \frac{\partial \chi}{\partial x} - t_i \frac{\partial u_i}{\partial x} \Big) dx\,dy$$

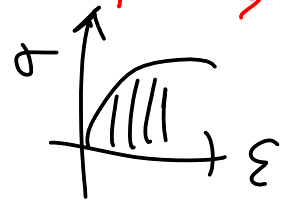
$$J = \iint \left( \frac{\partial \lambda}{\partial x} - t_i \frac{\partial u_i}{\partial x} \right) dx\, dy$$

$$\lambda = f(\varepsilon, r)$$


$$\begin{cases} t_i = \sigma_{ij}\, n_j \\ n_j = \dfrac{\partial}{\partial x_j} \end{cases}$$

$$\boxed{J = 0} \;!!$$

$$\lambda = \int \sigma\, d\varepsilon$$




$$\varepsilon = \frac{\partial}{\partial x}(u_{ij})$$

$$\varepsilon = \frac{\partial u}{\partial x} \ldots$$

$$\frac{\partial \lambda}{\partial x} = \frac{\partial \lambda}{\partial \varepsilon_{ij}} \frac{\partial \varepsilon_{ij}}{\partial x}$$

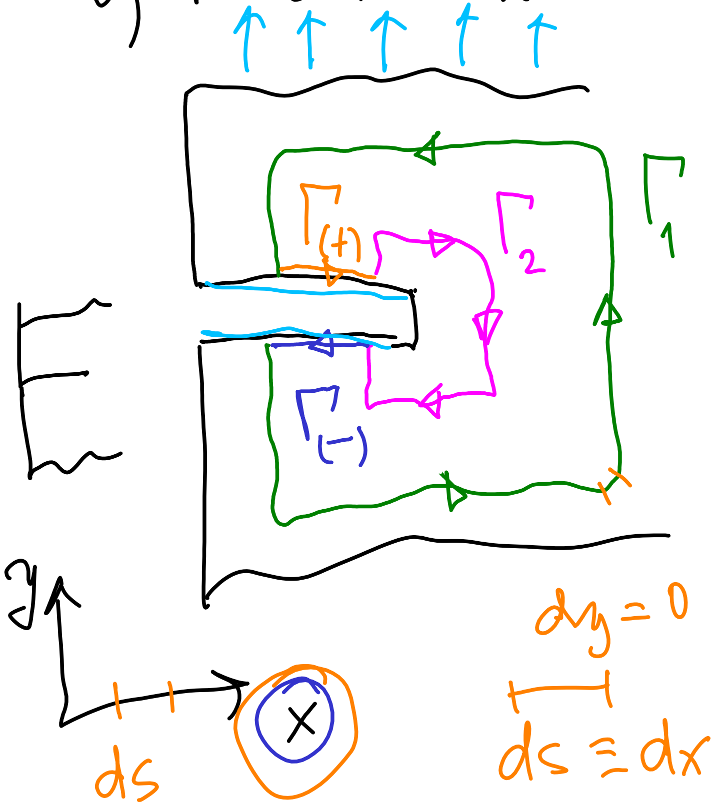
$$J = \iint \left( \frac{\partial \lambda}{\partial \varepsilon_{ij}} \frac{\partial \varepsilon_{ij}}{\partial x} - \sigma_{ij} \frac{\partial}{\partial x_j} \frac{\partial u_i}{\partial x} \right) dx\, dy$$

$$\sigma_{ij} = \frac{\partial \lambda}{\partial \varepsilon_{ij}}$$

$$\varepsilon_{ij} = \frac{\partial u_j}{\partial x_j}$$

$$= \iint \underbrace{\left( \sigma_{ij} \frac{\partial}{\partial x_j} \frac{\partial u_i}{\partial x} - \sigma_{ij} \frac{\partial}{\partial x_j} \frac{\partial u_i}{\partial x} \right)}_{0} dx\, dy$$

b) nezávisí na int. cestě:



$$J = J_1 + J_{(+)} - J_2 - J_{(-)} = 0$$

$$\text{III}: \quad J = \int \left( \chi \, \cancel{dy} - \cancel{t_i} \frac{\partial u_i}{\partial x} ds \right)$$

$dy = 0$ !

$ds = dx$

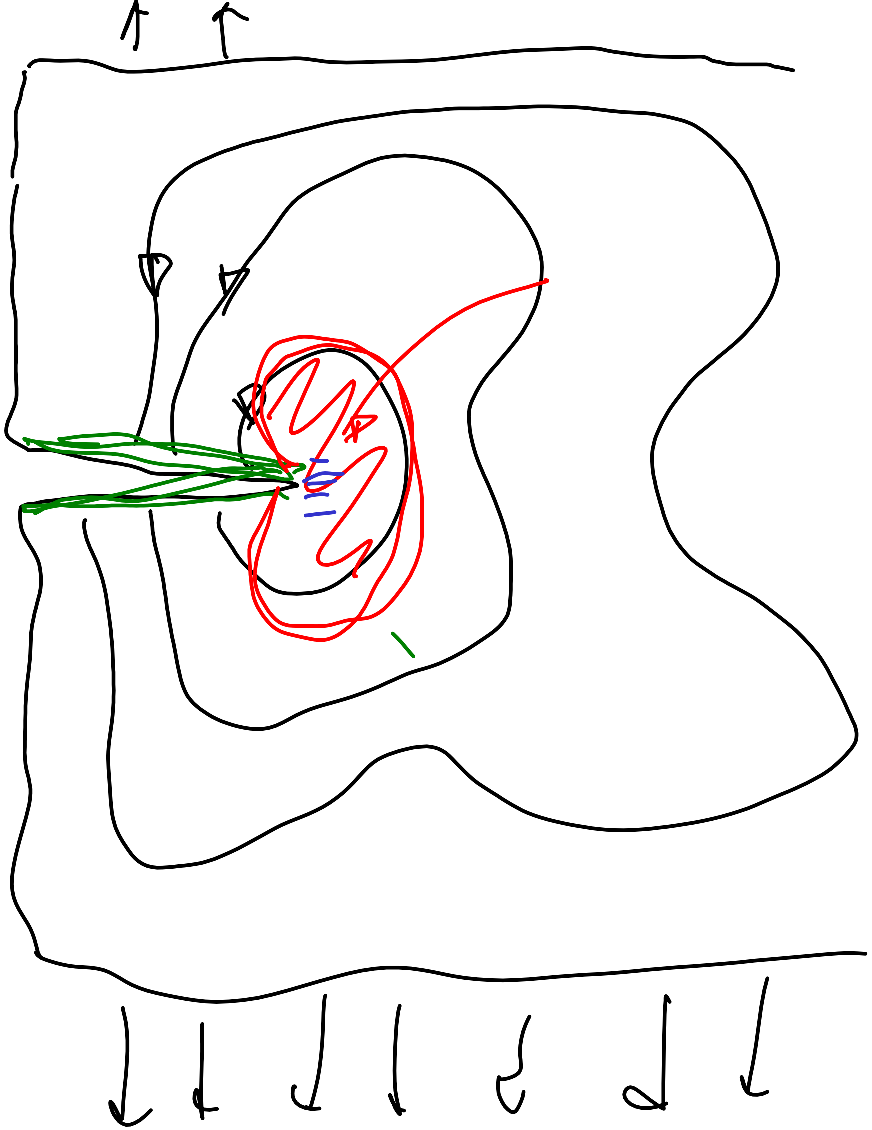
$ds \perp dy$

$dx$

tj. ... 0 ... protože jde o nezatížený okraj !!

- povrch. síly jsou nulové !!

$$J = J_1 - J_2 \Rightarrow \boxed{J_1 = J_2}\ !$$



$$J = \left| \int \ell \, dy - t_i \frac{\partial u_i}{\partial x} \, ds \right|$$

= stejné pro všechny cesty

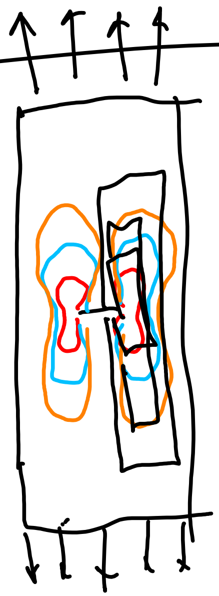
i pro nelin. mat.

i pro plast. mat.

pro elast. mat. :  $J \leftrightarrow G \leftrightarrow K$

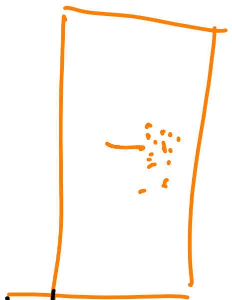
$$J = G = \begin{cases} \dfrac{K^2}{E} & \text{..... ROV. NAPJ } \sigma_z = 0 \\[2ex] \dfrac{K^2}{E}(1-\mu^2) & \text{.... ROV. DEF. } \varepsilon_z = 0 \end{cases}$$

$J_c$



$J_1 = J_2 = J_3$ (✓)

numerická SIMULACE

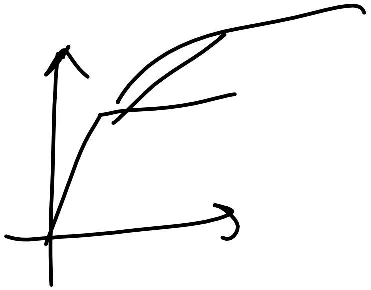


20

20

$$u = \begin{Bmatrix} u_x \\ u_y \end{Bmatrix}$$

$$\varepsilon_{ij} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$



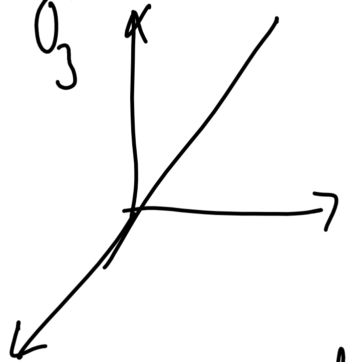
$\sigma \sim \varepsilon \sim u$

Hl. napětí $\rightarrow$ $I_1 \; I_2 \; I_3$ —

$J_1 \; J_2 \; J_3$ —

$\sim$ HMH (Rankin, Tresca, ...)

$f(\sigma_{ij}, k) = 0$




$\sigma = E\varepsilon$

$E_t$ ?

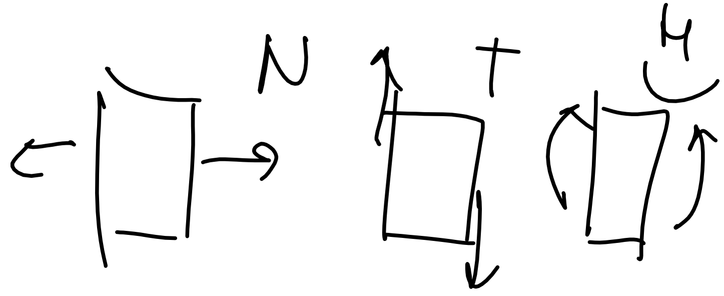
von Mises

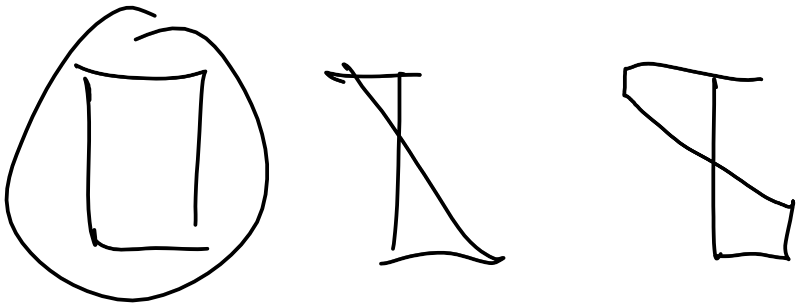
odvodit

$d\sigma = E\,d\varepsilon$

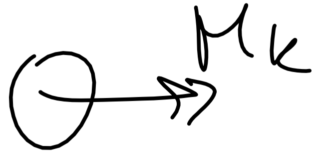
PPA → prvek, průřez

→ kce (SVK, SNK)

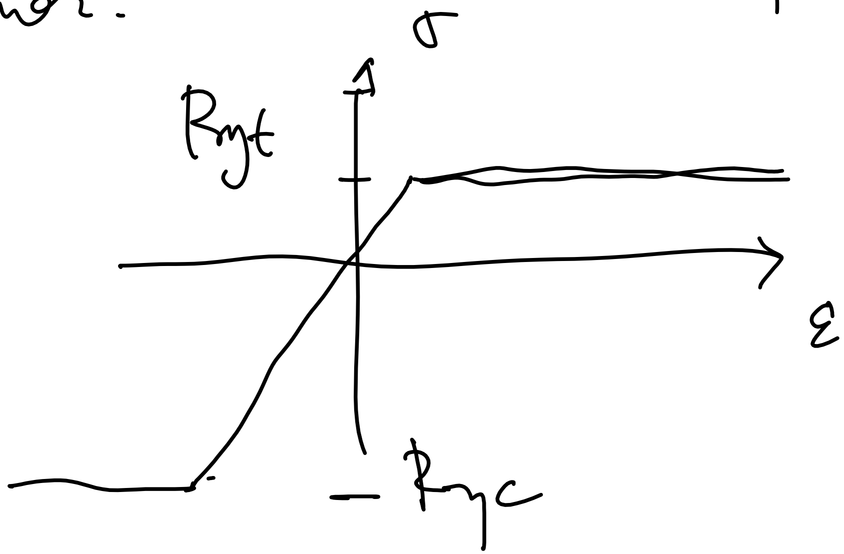


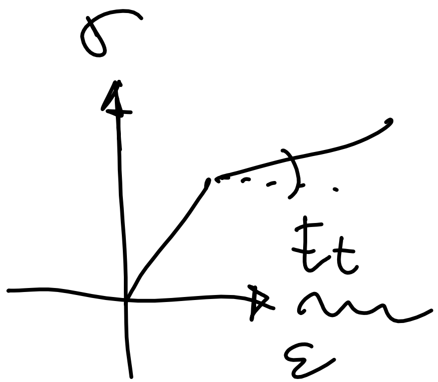
mor.



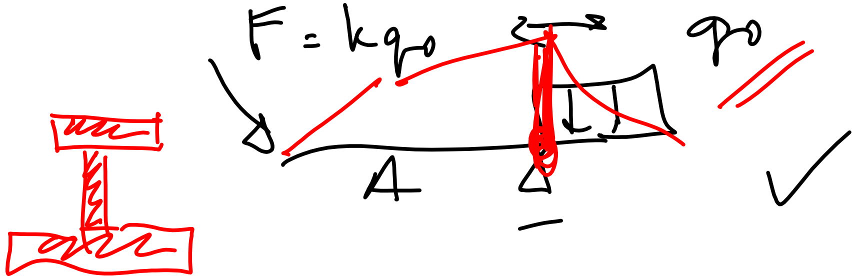

pl. rezerva průřezu!




!



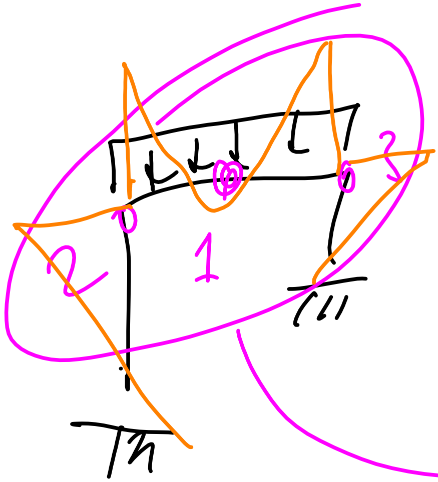
SVK



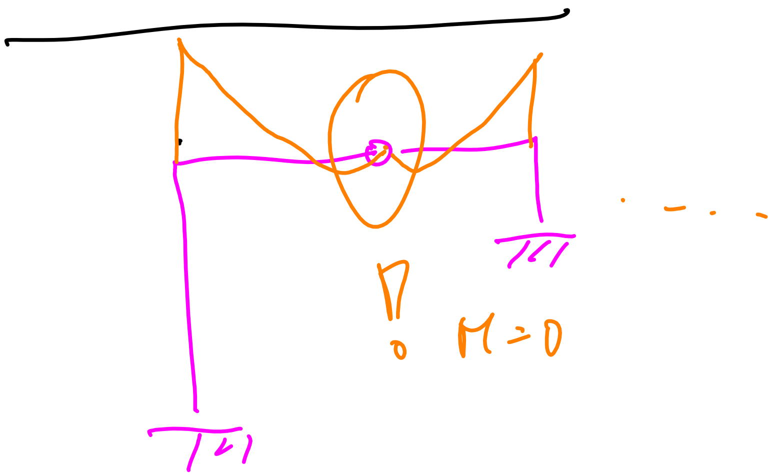

$q_0$

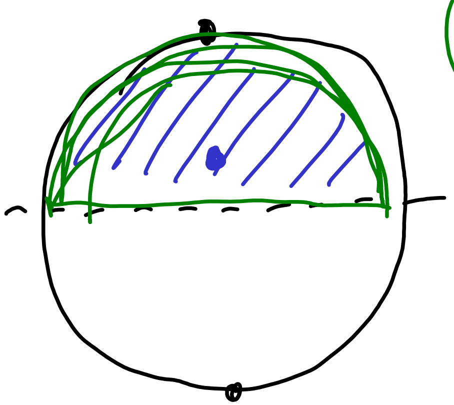
50% ?

$R_{yc} \neq R_{yt}$




plastick. metoda





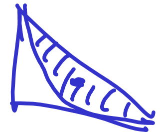
$W_{pl}$

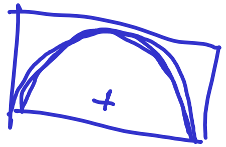
$$\xi = \frac{M_{pl}}{M_{el}} = \frac{W_{pl}}{W_{el}}$$

$\sigma = \frac{N}{A} \rightarrow (\sim 1{,}32$

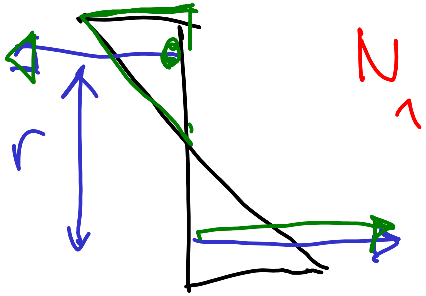
$N = \sigma \cdot A$

$N_1 =$

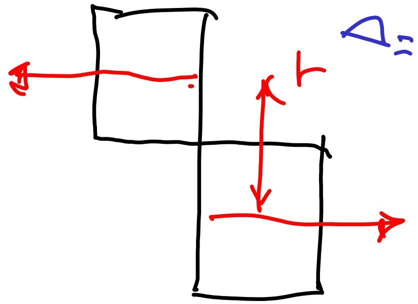




$\Delta = \frac{5}{6}$ ▭









$M_{pl} = N_1 r = N_2 r$

$\checkmark$ 2

$M_{el} = N_1 r = N_2 r$

$\int \Delta$ ⌓