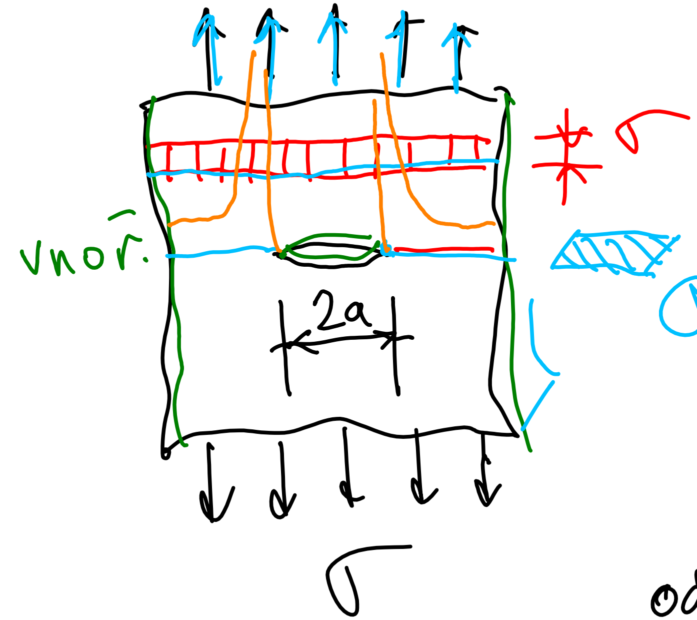
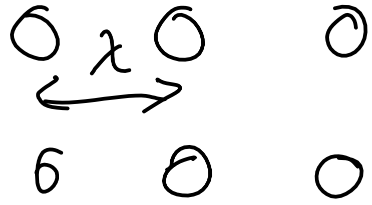
2D ⟨ RN, RD

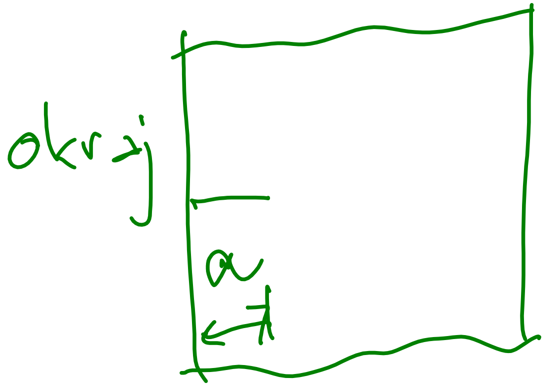
$\left(\varepsilon = \dfrac{x}{a_0} \quad \left(\dfrac{\Delta l}{l} \right) \right)$

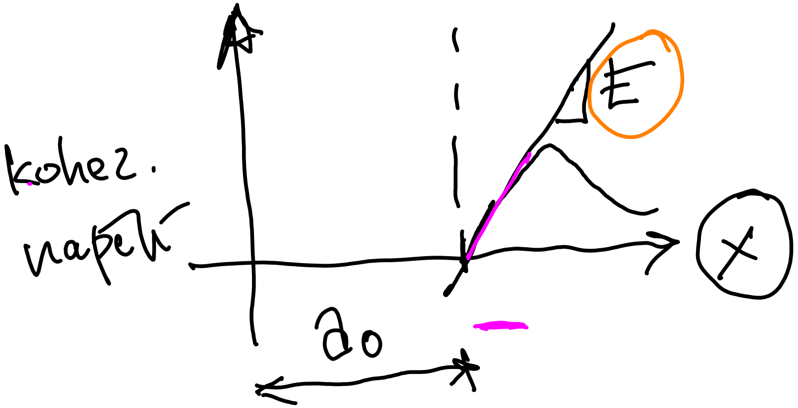


$\sigma = \dfrac{F}{A}$

Teoretická kohez. pevnost







$\sigma = \sigma_c \cdot \sin \dfrac{2\pi x}{x}$

$\sin x = \alpha$

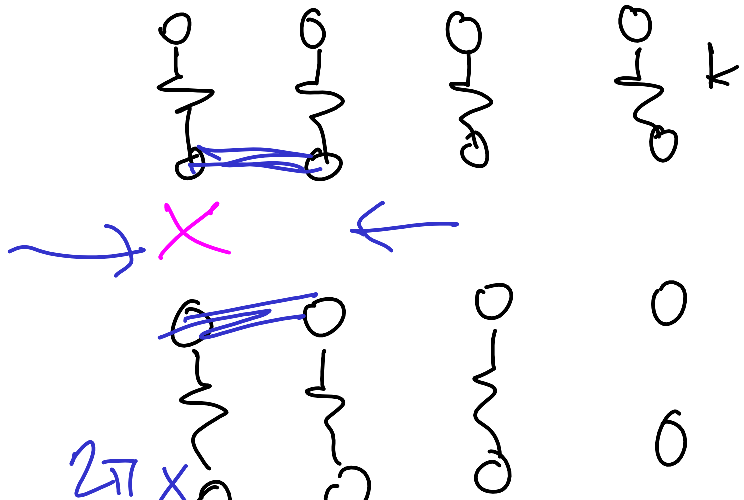
$\sigma = \sigma_c \cdot \dfrac{2\pi x}{x}$

$\sigma = E \varepsilon$

$$\sigma = \sigma_c \frac{2\pi x}{\lambda}$$

$$\sigma = E\varepsilon = E \frac{x}{a_0}$$

$$\sigma_c \frac{2\pi \cancel{x}}{\lambda} = E \frac{\cancel{x}}{a_0}$$



$$\sigma_c = \frac{\lambda E}{2\pi a_0}$$

$$2\gamma_s = \int_0^{\lambda/2} \sigma \, dx = \int_0^{\lambda/2} \sigma_c \sin \frac{2\pi x}{\lambda} \, dx =$$

$$\sigma_c \cdot \left[-\frac{\lambda}{2\pi} \cdot \cos \frac{2\pi x}{\lambda} \right]_0^{\lambda/2} = \sigma_c \cdot \frac{\lambda}{2\pi}$$

$$\sigma_c = \frac{x E}{2\pi a_0} = \frac{2\pi \gamma_s E}{2\pi a_0 \sigma_c} = \frac{\gamma_s E}{a_0 \sigma_c}$$

$$\sigma_c = \sqrt{\frac{\gamma_s \cdot E}{a_0}}$$

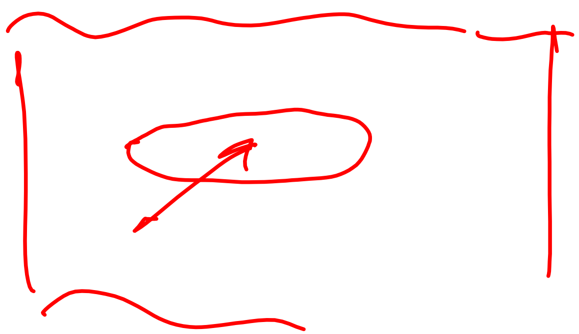
Koy: $\boxed{\sigma_c \sim \frac{1}{10} \div \frac{1}{5} E}$!

$E_{ocel} = 210 \cdot 10^3$ MPa

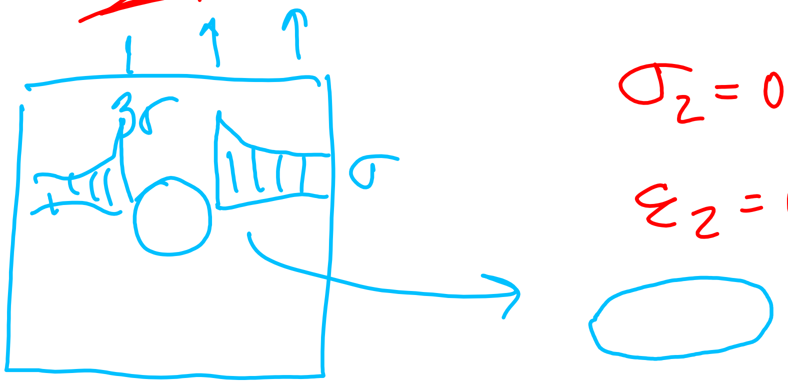
ROZPOR !

→ ∃! a priori trhlina

→ rozbor napjatosti



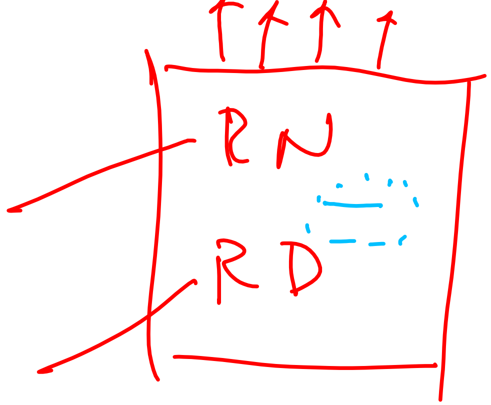
2D, 3D



2D

$\sigma_z = 0$

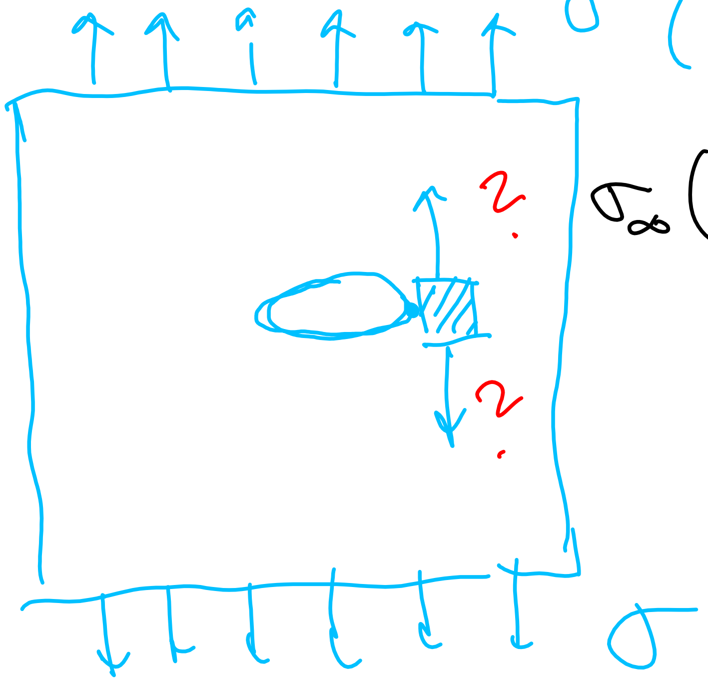
$\varepsilon_z = 0$



$\sigma_x(x,y) =$

$\sigma_y(x,y) =$

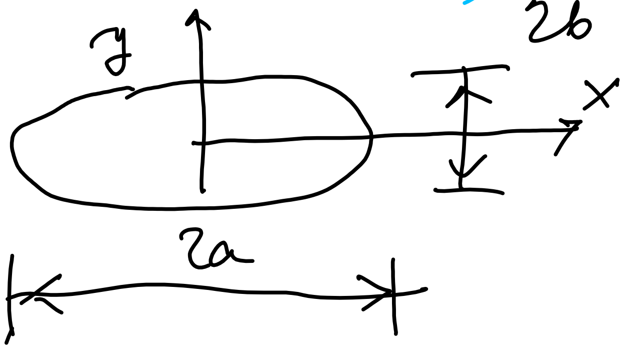
$\tau_{xy}(x,y) =$



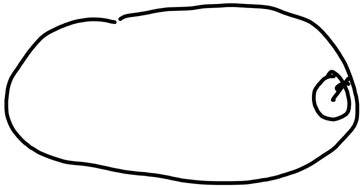
$\sigma \; (\sigma_\infty)$

$\sigma_\infty \left(1 + 2\frac{a}{b}\right)$

Inglis (1913)



$$\sigma_{max} = \sigma_\infty \left(1 + 2\frac{a}{b}\right)$$

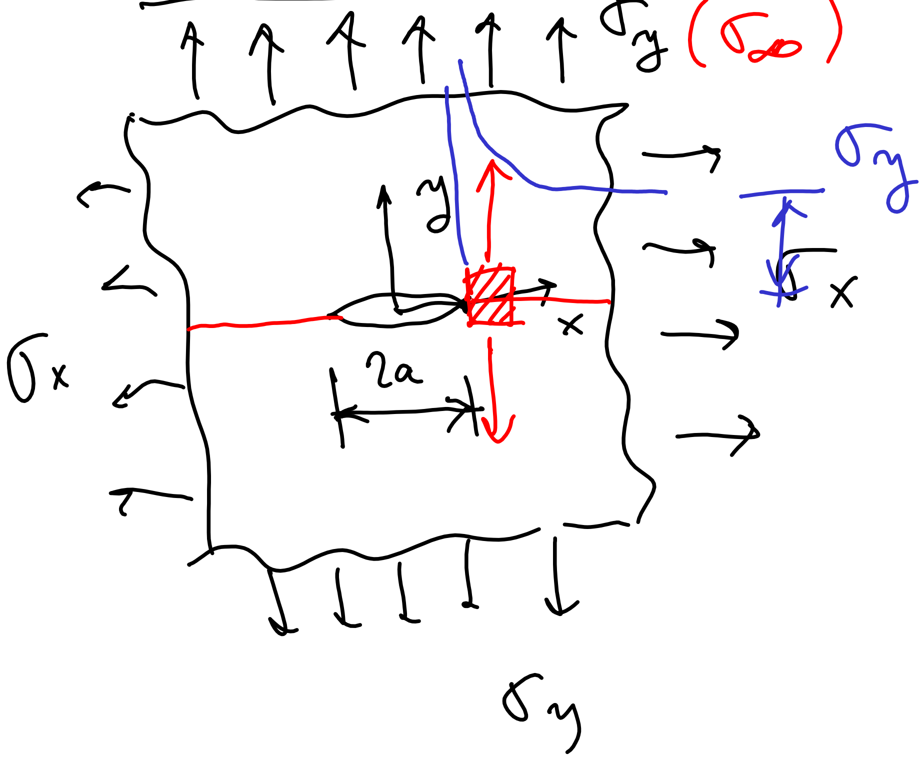


$$\rho = \frac{b^2}{a} \qquad b = \sqrt{a\rho} \qquad \text{Inglis, 1913?}$$

$$\sigma_{max} = \sigma_\infty \left(1 + 2\frac{a}{\sqrt{a\rho}}\right)$$
$$= \sigma_\infty \left(1 + \sqrt{\frac{a}{\rho}}\right)$$

Westergaard, 1939

- trhlina, ne elipsa

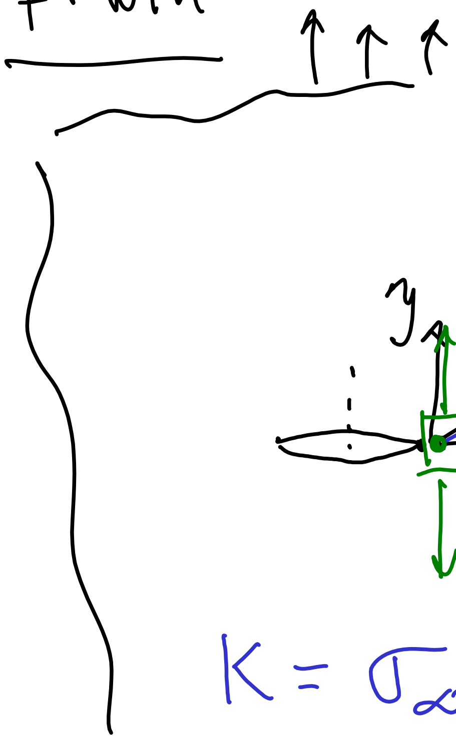


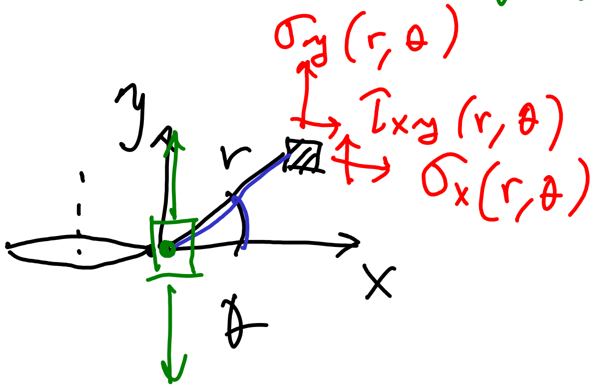
Airy fce napětí

$$\frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = 0$$

$$\sigma_y = \frac{\sigma_\infty}{\sqrt{1 - \left(\frac{a}{x}\right)^2}}$$

Irwin:



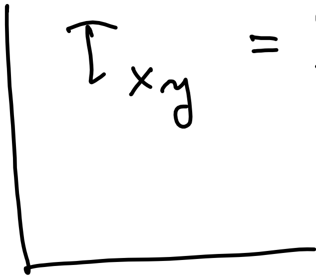


lim
$r \to 0$
$\theta = 0$

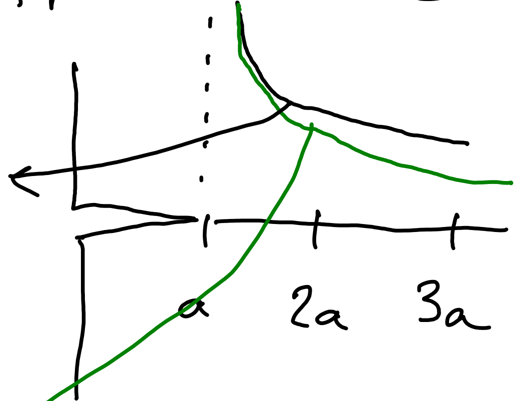
$$\sigma_y = \frac{\sigma_\infty \sqrt{\pi a}}{\sqrt{2\pi r}} \cdot \cos\frac{\theta}{2}\left(\cancel{1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}}\right) \quad 1$$

$$\sigma_x = \frac{\sigma_\infty \sqrt{\pi a}}{\sqrt{2\pi r}} \cdot \cos\frac{\theta}{2}\left[1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right.$$

$$\tau_{xy} = \frac{\sigma_\infty \sqrt{\pi a}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2}$$



$$K = \sigma_\infty \sqrt{\pi a}$$



Inglis:

$$\sigma_y = \frac{\sigma_\infty}{\sqrt{1-\left(\frac{a}{x}\right)^2}}$$

Irwin:

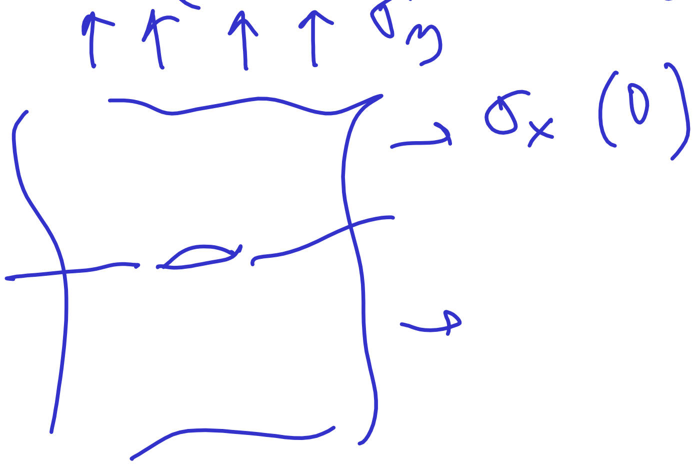
$$\sigma_y = \frac{\sigma_\infty \sqrt{\pi a}}{\sqrt{2\pi r}}$$

$$\sigma_x = \frac{K}{\sqrt{2\pi r}} \cdot f(\theta) \qquad \sigma_y = \frac{K}{\sqrt{2\pi r}} \cdot g(\theta) \qquad \tau_{xy} = \frac{K}{\sqrt{2\pi r}} h(\theta)$$

$$\sigma_y(r,\theta)$$

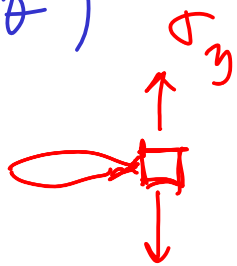
$$\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

→ (rozbor napjatost ; < KSS (x,y), PSS (r,θ)

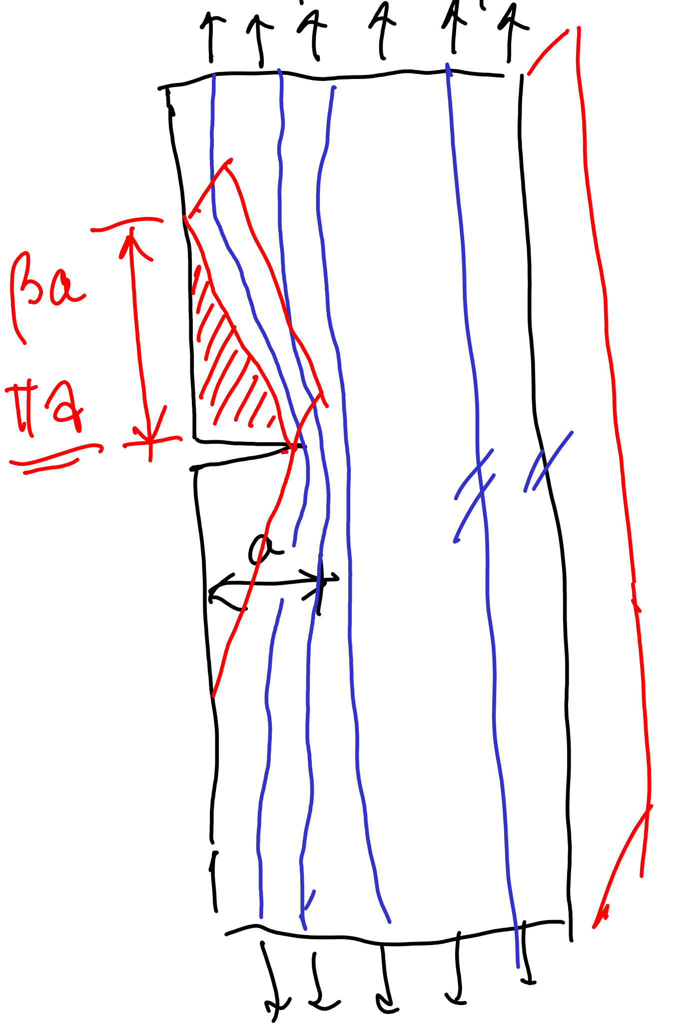


$$\sigma_x(r,\theta)$$

$\lim$
$r \to 0$
$\theta = 0$

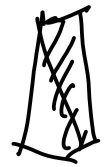


Griffith, 1930 : Energeticky



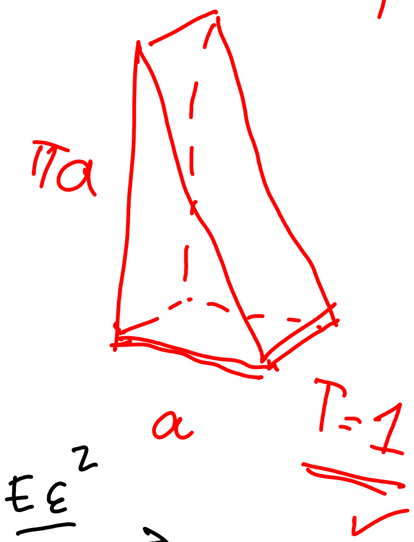
? Uvolněná energie

$U = ?$

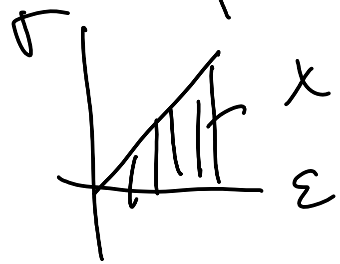


$$U = \int \lambda \, dV = -\lambda \cdot V$$

$$V = \frac{1}{2}\pi a^2 \cdot 1$$



$$= -\lambda \, \frac{1}{2}\pi a^2$$



$$\lambda = \int \sigma \, d\varepsilon = \overset{LEM}{=} \frac{1}{2}\sigma\varepsilon = \frac{E\varepsilon^2}{2} = \frac{\sigma^2}{2E}$$

$$U_{\Delta} = -\frac{\sigma^2}{2E} \cdot \frac{1}{2}\pi a^2 = -\frac{\sigma^2}{4E}\pi a^2$$

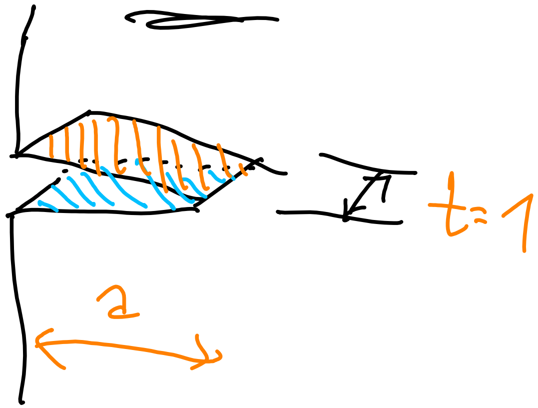
energie uvolněné z mat. při vzniku trhliny délky a:

$$U = 2U_{\Delta} = -\frac{\sigma^2}{2E}\pi a^2$$ // na "1" tloušťky

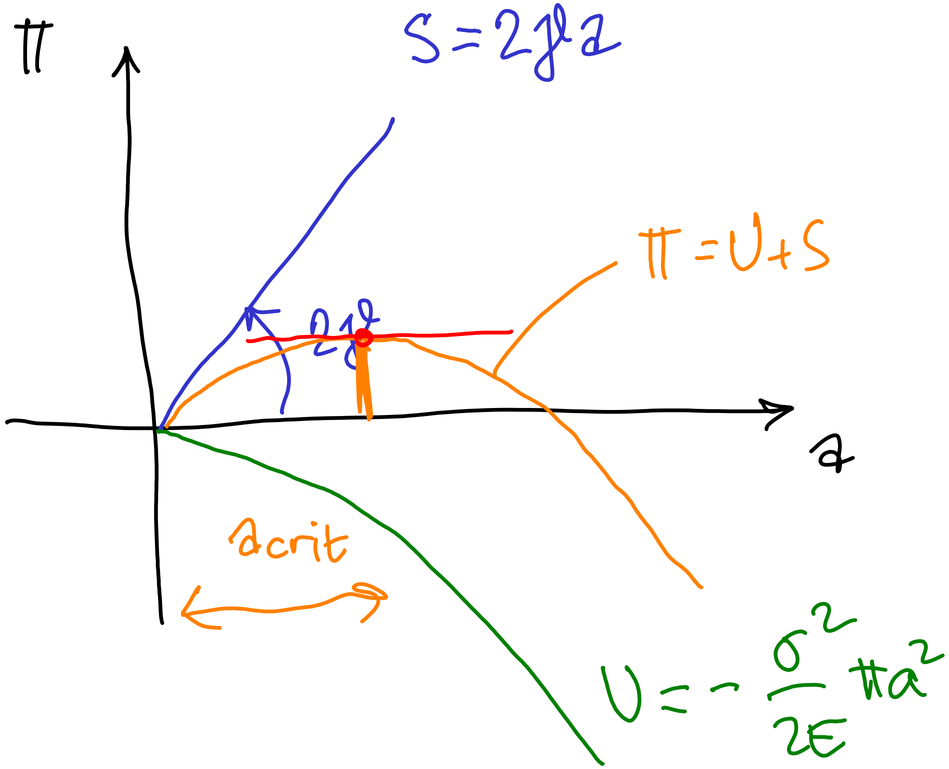
Povrchová energie S

γ - povrchová energie $[J/m^2]$

$$S = \int \gamma \, dA = \gamma \cdot A = \gamma \cdot 2a \cdot 1$$



$$\Pi = U + S = 2\gamma a - \frac{\sigma^2}{2E}\pi a^2$$



Π ✓ $\quad \Pi = \Pi(a)$

$$\frac{\partial \Pi}{\partial a} = 0 \longrightarrow a_{crit}$$

$$\frac{\partial \Pi}{\partial a} = 2\gamma - \frac{\sigma^2}{E}\pi a = 0$$

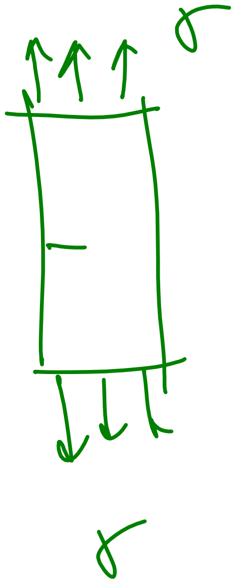
$\frac{\partial \Pi}{\partial a} = 0$: $\quad 2\gamma - \frac{\sigma^2}{E}\pi a = 0$

mater

$$\boxed{a_{crit} = \frac{2\gamma E}{\pi \sigma^2}}$$

Griffith energ. kritérium

půs. napětí



$$\sigma_f = \sqrt{\frac{2\gamma E}{\pi a}}$$

LOMOVÉ NAPĚTÍ pro přísluš. délku trhliny a.

PLATNOST: křehké mat. & LLM! $\quad \sigma = E\varepsilon$

$$\boxed{\sigma < \sigma_y}$$

Irwin (1950) modifikace Griffith

pro tvárné mat:

$$\gamma = \underline{\gamma_s} + \underline{\gamma_p}$$

γ_p ... plastická disipace

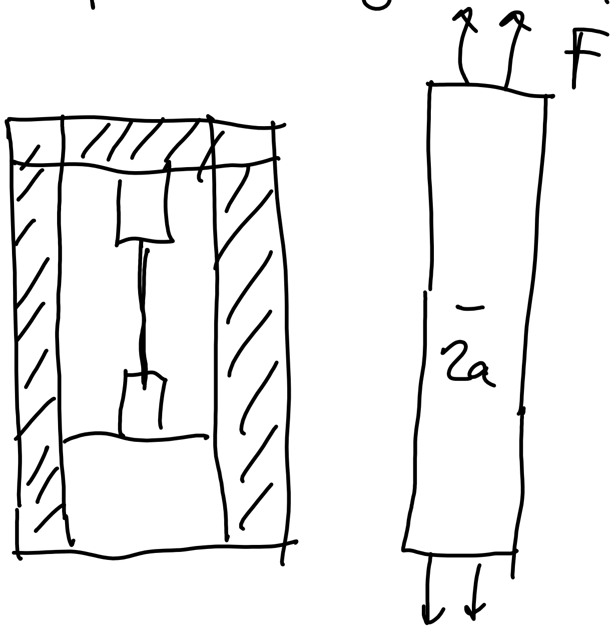
$\gamma_s = 1\text{-}2\ \mathrm{J/m^2}$ sklo

$\gamma_p = 1000\ \mathrm{J/m^2}$ ocel

$$G = \frac{\partial U}{\partial a}$$

$$\frac{dS}{da} = \frac{\partial(2\gamma a)}{\partial a} = 2\gamma = 2(\gamma_s + \underline{\gamma_p})$$

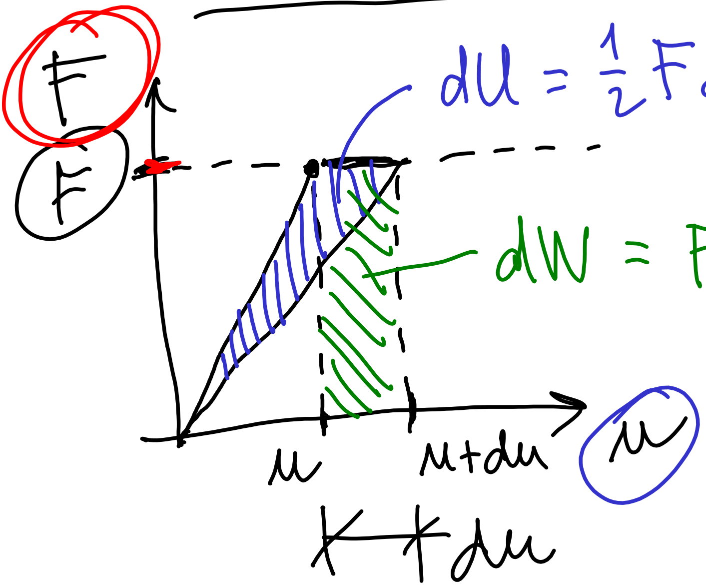
Experimenty. Rychlost uvolňování energie



řízení → silou (σ)
→ posunutí (ε)

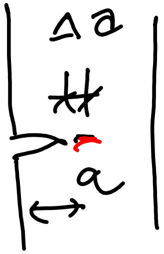
$$\sigma = \frac{F}{A} \qquad \varepsilon = \frac{\Delta l}{l}$$

a) řízení silou



$dU = \frac{1}{2} F\,du$

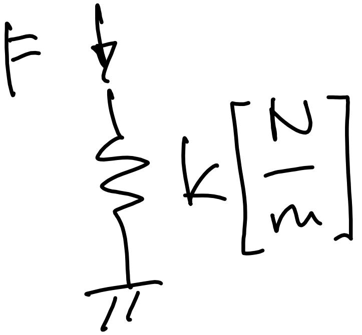
$dW = F\,du$



$$U = \int_0^u F\,du = \frac{1}{2} F u$$

$$W = F u$$

$$\Pi = U - W = \frac{1}{2} F u - F u = -\frac{1}{2} F u$$



poddajnost $= \dfrac{u}{F} \rightarrow C = \dfrac{u}{F}$

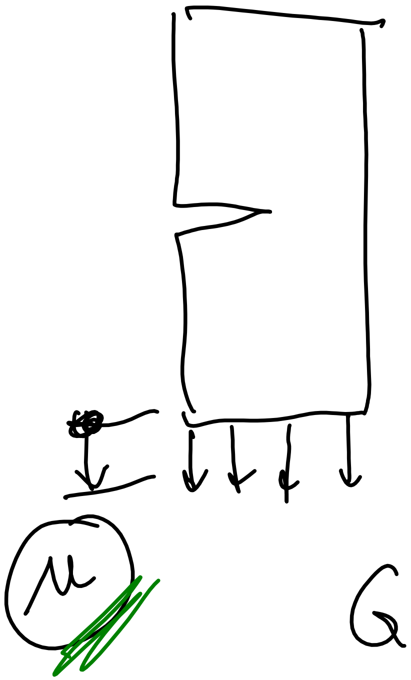
$F = k u$

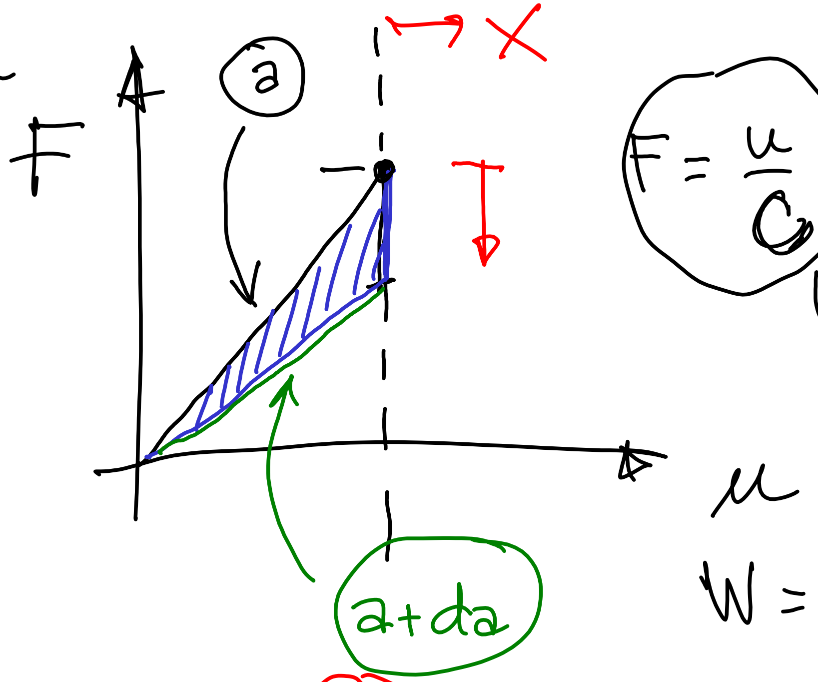
$\delta = \frac{1}{k} \; (= C)$

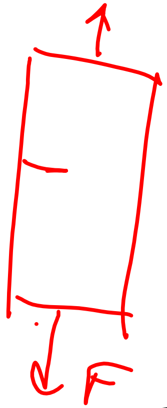
$$\Pi = -\frac{1}{2} F u = -\frac{1}{2} F \cdot F C = -\frac{1}{2} F^2 C$$

$$G = -\frac{1}{t} \frac{\partial \Pi}{\partial a} = -\frac{1}{t} \frac{\partial}{\partial a}\left(-\frac{1}{2} F^2 C\right) = \boxed{\frac{1}{2t} F^2 \frac{\partial C}{\partial a}}$$

b) řízení posuvem







$$F = \frac{u}{C}$$

$$u = FC$$

$$W = 0$$

$$\pi = U - W = U$$

$$U = \frac{1}{2} F u$$

$$\partial (F u)$$

$$\frac{\partial F}{\partial a} \cdot u + \cancel{\frac{\partial u}{\partial a}} \cdot F$$

$$G = -\frac{1}{t}\frac{\partial \pi}{\partial a} = -\frac{1}{t}\frac{\partial U}{\partial a}$$

$$G = -\frac{1}{t} \cdot u \cdot \frac{\partial F}{\partial a} \qquad \frac{dF}{da} = \frac{u}{C^2}\frac{dC}{da}$$

$$G = -\frac{1}{t} \cdot u \cdot \frac{u}{C^2}\frac{dC}{da} = \boxed{-\frac{1}{t} F^2 \frac{dC}{da}}$$